ME 4555 – Lecture 2 – Mechanical systems I

The next several lectures focus on mathematical modeling.

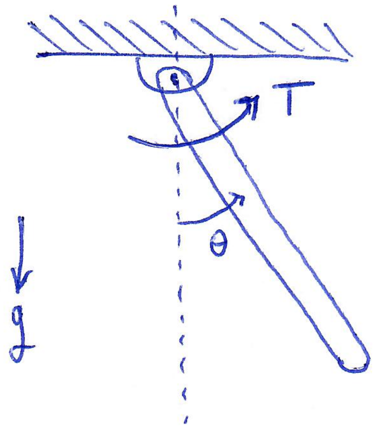
Turning a physical system into a mathematical model.

$$\left\{\begin{array}{l}\text{mechanical system}\\ \text{electrical system}\\ \text{electromechanical}\\ \text{hydraulic}\\ \ldots\end{array}\right\} \longrightarrow \left\{\begin{array}{l}\text{ordinary differential}\\ \quad\text{equations (ODEs)}\\ \text{algebraic equations}\end{array}\right\}$$

No model is perfect! A model should only be as complicated as it needs to be for the application.

- ☆ too simple and it will not accurately represent reality and will lead to bad predictions + control.
- ☆ too complex/realistic and it may become intractable to solve without any practical improvement in fidelity.

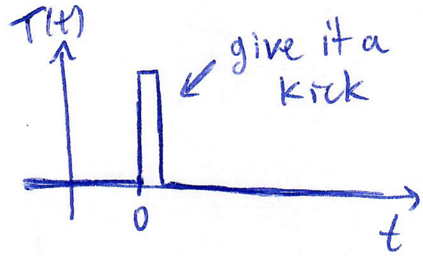
<u>Ex</u>: pendulum hanging from the ceiling. Let's think about what the most realistic possible model might look like, and what simplifications/assumptions we should make.

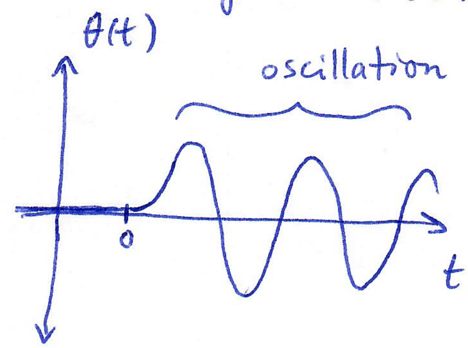


$\theta(t)$: angle of pendulum (output).

$T(t)$: applied torque (input)

We might expect this sort of behavior:





What might we assume in our model?

1) material is somewhat <u>rigid</u> (eg. if it's made of metal or wood).
it won't bend, stretch, or otherwise deform

2) pendulum is heavy compared to the air mass surrounding it,
so there is <u>no air drag/resistance</u>.

3) hinge is <u>frictionless</u>, if e.g. time horizon is sufficiently short.

4) speed is relatively slow (no relativistic effects)

5) neglect earth effects; gravity is constant, no e.g. Coriolis force
or other forces due to earth spinning.

6) angle θ is relatively small, so $\sin\theta \approx \theta$.

<u>Today</u> : Mechanical systems with <u>translational</u> motion (movement in one direction)

Most important fact : Newton's 2^{nd} law

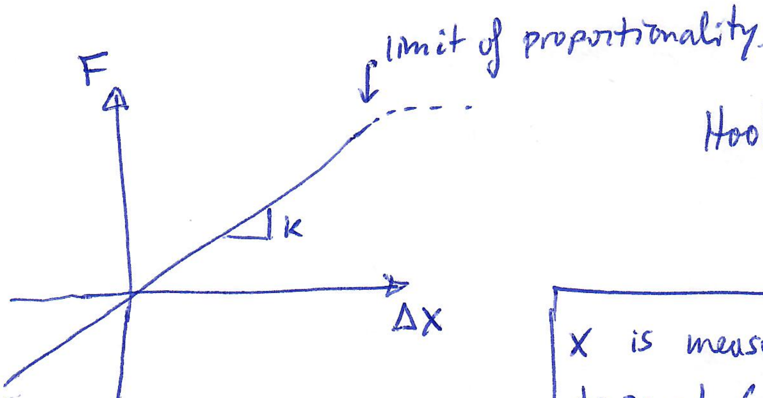
$$\underset{\text{mass of object}}{m}\;\underset{\text{acceleration of object}}{\ddot{x}} = \underbrace{\sum F}_{\text{sum of external forces acting on the object}} \qquad x(t) = \text{position.}$$

$$\left(\ddot{x} = \frac{d^2x}{dt^2}\right)$$

we will work in 1-D for now. (all forces & motion in one direction.

<u>Note</u> : Newton's 2^{nd} law is only true if we are in an inertial frame (non-accelerating).

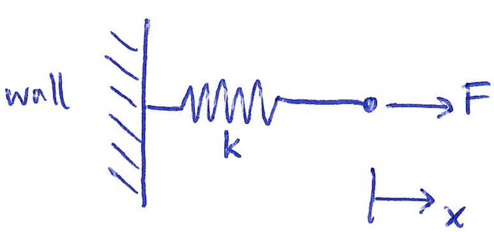
Stiffness <u>elements</u> (springs) store energy by deforming.



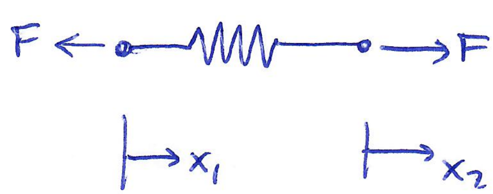
Hooke's law : $\underbrace{F}_{\substack{\text{force}\\ \text{(Newtons)}}} = \overbrace{k}^{\text{spring constant (N/m)}} \cdot \underbrace{\Delta x}_{\substack{\text{deformation}\\ \text{(can be negative!)}\\ \text{(meters)}}}$

$\boxed{\text{x is measured } \underline{\text{relative}} \text{ } \underline{\text{to zero}}! \text{ (zero x = zero force)}}$

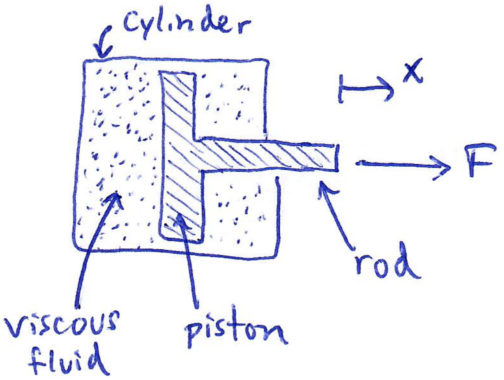
<u>ex 1</u> : $F = kx$



<u>ex2</u> $F = k(x_2 - x_1)$



Friction elements (dampers or dashpots) dissipate energy due to motion.



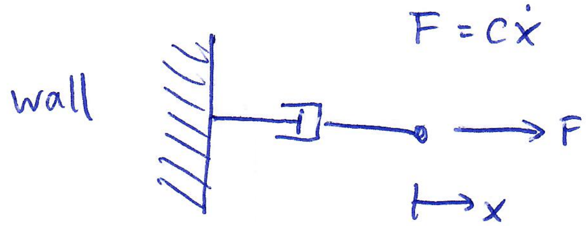
damping coefficient $\left(\frac{N \cdot s}{m}\right)$

$$\underbrace{F}_{\text{force (N)}} = c\,\underbrace{\dot{x}}_{\text{velocity (m/s)}}$$

interpretation, if you move slowly, you move easily (little resistance)

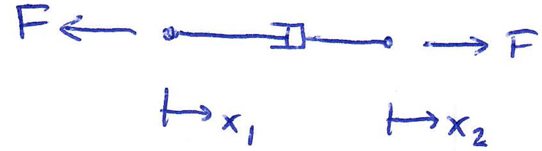
ex 1:

$F = c\dot{x}$

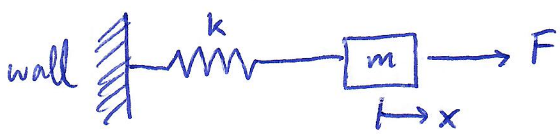


ex 2:

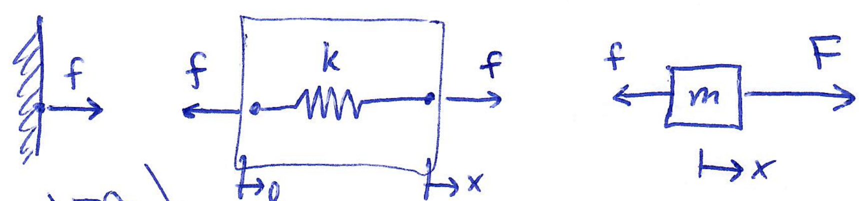
$F = c(\dot{x}_2 - \dot{x}_1)$



For interconnected systems, separate into components and draw a free body diagram.



Newtons 3rd law:
if A exerts a force on B, B exerts equal + opposite force on A.



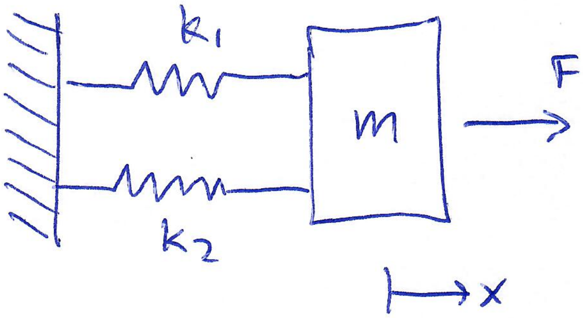
(reaction force from wall.)

$f = kx$

$F - f = m\ddot{x}$

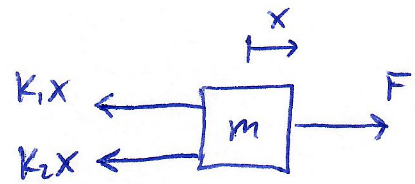
$$\boxed{m\ddot{x} + kx = F.}$$

ex: springs in parallel:



Note: assume 1-D motion (springs don't cause twisting).

FBD:



(sum of forces) $= m\ddot{x}$

$$\Rightarrow \quad (F - k_1 x - k_2 x) = m\ddot{x} \quad \Rightarrow \quad \boxed{m\ddot{x} + (k_1+k_2)x = F}$$

$$K_{eq} = k_1 + k_2$$

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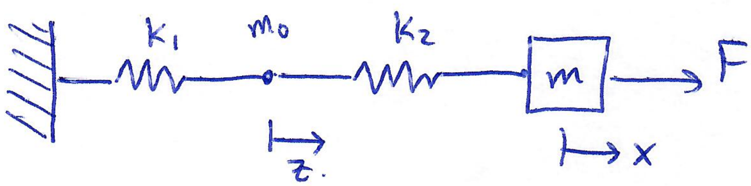
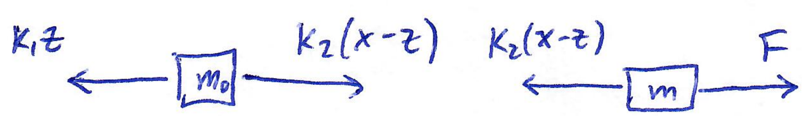
ex: springs in series:



★ insert fictitious mass (zero mass) in between both springs. Call it m_0.

FBD:



$$\boxed{m_0\ddot{z} = k_2(x-z) - k_1 z} \qquad \boxed{m\ddot{x} = F - k_2(x-z)}$$

if $m_0 = 0$ then $k_1 z = k_2(x-z)$

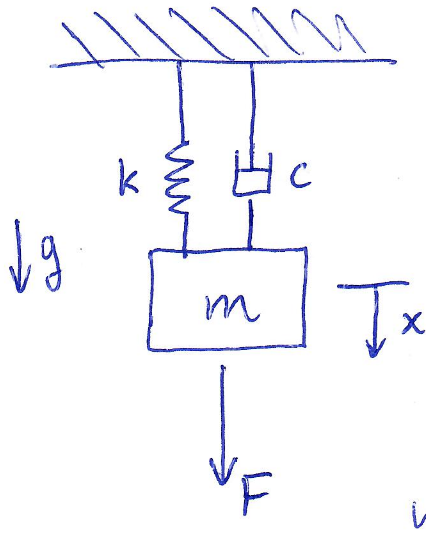
solve for z: $z = \dfrac{k_2}{k_1+k_2}x$.

substitute: $m\ddot{x} = F - k_2\left(x - \dfrac{k_2}{k_1+k_2}x\right)$

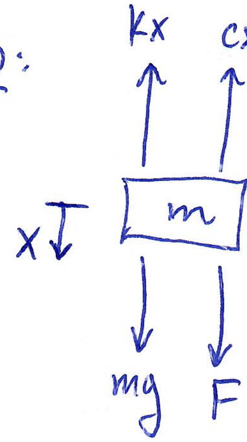
$$\Downarrow$$

$$K_{eq} = \frac{k_1 k_2}{k_1+k_2} \quad \text{or} \quad \frac{1}{K_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Longleftarrow \quad \boxed{m\ddot{x} + \frac{k_1 k_2}{k_1+k_2}x = F}$$

Ex: gravity!



FBD:



Newton's 2^{nd} law:

$$m\ddot{x} = (F + mg - kx - c\dot{x})$$

$$\Downarrow$$

$$m\ddot{x} + c\dot{x} + kx = F + mg.$$

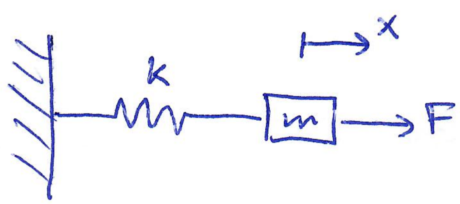
We can "cancel" gravity: Let $\boxed{\tilde{x} = x - \frac{mg}{k}}$

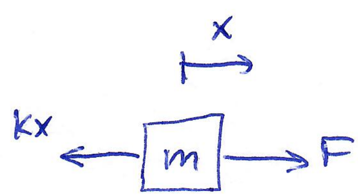
← shift in equilibrium position.

substituting, we obtain $\underline{m\ddot{\tilde{x}} + c\dot{\tilde{x}} + k\tilde{x} = F}$

equations of motion (EOM)

Ex: flipping signs:

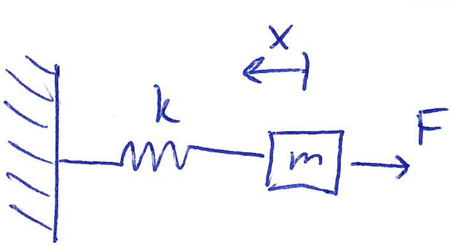


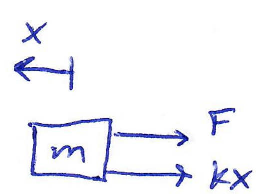


(if $x > 0$, k "pulls" on mass)

$$m\ddot{x} = (F - kx) \Rightarrow \boxed{m\ddot{x} + kx = F}$$

(right is positive)





(if $x > 0$, k "pushes" mass to the right)

$$m\ddot{x} = (-F - kx) \Rightarrow \boxed{m\ddot{x} + kx = -F}$$

(left is positive)